Asset Allocation in the Presence of Uncertainty

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- Return, risk and correlation estimates are fraught with uncertainty (the inability to know whether our estimates are correct, even if our method of estimation is unbiased).
- Diversification, while always desirable, is especially valuable in the presence of uncertainty. In fact, uncertainty can render diversification a more “optimal” solution than traditional mean-variance optimization.
- This is why we pursue diversification in all market environments and why we maintain balanced portfolios, regardless of recent returns.

“When I bet on horses, I never lose. Why? I bet on all the horses.”
Tom Haverford, NBC sitcom Parks and Recreation

Misguided though it may be, Tom’s thinking is not without its wisdom. We in the investment world might benefit from thinking twice about his statement. We know we’re supposed to diversify our assets. We’ve been told time and time again that this is the only way to make our portfolios truly efficient. Still, the temptation is strong to abandon diversification in search of higher returns, particularly when certain concentrated portfolios have performed so well in recent memory and when we think we can predict severe disparities between returns on different asset classes. The current market is a case in point, as many investors consider it a foregone conclusion that stocks will outperform bonds in the near term because current yields are so low in historical terms. Given this assumption, should we really allocate a meaningful portion of our portfolios to fixed income?

While we, too, expect stocks to fare better than bonds in the months ahead, we still believe bonds play a crucial role in portfolio diversification—even during “low yield” environments. On a similar note, diversification also entails managing macroeconomic risk exposures, e.g., protecting the purchasing power of one’s portfolio even when an inflation outbreak is not one’s base case expectation.

To highlight our logic, we explore the advantages of diversification and, perhaps more importantly, the dangers associated with diversifying insufficiently. We consider two different methods of portfolio construction and evaluate their merits and vulnerabilities.

The first method, known as mean-variance optimization (MVO), relies explicitly on return forecasts and more closely resembles the strategy of the investor mentioned above who seeks higher risk-adjusted returns by allocating more heavily to asset classes that are forecast to perform better. The second, referred to here as “maximum diversification (MD),” ignores return estimates and instead seeks only to take full advantage of the risk-reduction benefits afforded by blending assets that exhibit low correlations to each other.

We show that, while MVO does account for risk and seeks to minimize variance for a given level of return, it is often too sensitive to error-prone estimates of expected return. In the presence of uncertainty, particularly in return forecasts, we show that diversification can often beat MVO on its own terms—namely, risk-adjusted return.

Importantly, the method of diversification that we chose is merely an illustration—we are not presenting this approach as the indisputably superior method of diversifying a portfolio. Instead, we seek only to demonstrate the importance of devoting explicit attention to portfolio diversification in general. Our analysis sheds light on some crucial tenets of our investment philosophy and explains specifically why we pursue diversification even when it is an unpopular decision.
A Refresher on Portfolio Theory

Before exploring our analysis, it is useful to recall some important concepts of traditional portfolio theory. For investors seeking higher returns with lower risk (volatility), modern portfolio theory recommends a method known as mean-variance optimization. This method takes a given set of assumptions for the returns, volatilities, and correlations across asset classes and solves for the asset class weights that maximize a portfolio’s return for a given level of risk (or, equivalently, minimize portfolio risk for a given level of return). In Exhibit 1, this is known as the efficient frontier.

When investors have access to a risk-free asset, the Sharpe Ratio becomes crucial to this analysis: this measure equals the portfolio’s expected excess return over the risk-free rate divided by the portfolio’s standard deviation. Investors now have a choice of how they allocate to the risk-free asset and the optimal risky portfolio, with varying combinations represented by the capital allocation line (CAL). The point where the efficient frontier and the CAL meet is known as the mean-variance optimal portfolio, which is the combination of risky assets with the highest possible Sharpe Ratio. Since investors can (by assumption) borrow and lend at the risk-free rate, they can achieve the maximum available Sharpe Ratio at whatever level of risk or return they desire by blending the optimal “risky” portfolio with borrowing or lending at the risk-free rate.

Exhibit 1: Mean-Variance Optimization

Bring in the Noise

Several vulnerabilities of MVO are well known in the investment industry. Most notably, the method’s output is rather sensitive to small changes in its inputs; a small modification of an asset class’s expected return can significantly alter its allocation as recommended by MVO. In similar fashion, MVO tends to lead to relatively concentrated portfolios. Since it seeks the highest return for a given level of risk, it will often forego diversification and concentrate portfolio exposures in a few high-returning asset classes. Under the assumptions entered into the optimization, the benefit outweighs the cost, and expected risk-adjusted return is enhanced by allocating in this concentrated fashion.

Unfortunately, these inputs are, as mentioned, assumptions. In the capital markets, we cannot be sure that our estimates of expected returns, risks and correlations are perfectly accurate, even over long periods of time. Central to this reality is the distinction between risk and uncertainty. MVO seeks to account fully for the risk inherent in asset returns, but it necessarily ignores the uncertainty surrounding their distributions. Let us explore the distinction between risk and uncertainty with an example.

Investors are generally familiar with the concept of risk. Asset returns are driven by a wide variety of factors, such as economic growth, inflation and real interest rates, and the development of these factors over time is widely considered to be unknown today. We have expectations for asset class returns, but we know that our expectations will not necessarily be met over any given time horizon. Risk can be represented (albeit simplistically) by the rolling of 6-sided dice. While we cannot know which number will be rolled, we know all of the possible outcomes and their associated probabilities of occurring (namely, numbers 1-6, each with 1/6 probability for each die rolled). If someone offers us a wager based on the rolling of dice, we know the expected value of the wager and the risk associated with the randomness of the roll.

In contrast, uncertainty arises in situations where the distribution of possible outcomes cannot be known in advance. In the dice example, uncertainty would be present if we did not know the number of sides on the dice or the probabilities associated with each number being rolled. We can develop our best estimate of the distribution of possible outcomes, but we cannot know for certain that our estimate is accurate. This situation more closely mirrors the position in which investors in the capital markets find themselves. We can use historical data, economic intuition, quantitative reasoning, and a variety of other tools to forecast expected asset class return distributions, but we will almost invariably be uncertain as to whether or not these distributions are correct representations of reality.
Perhaps the simplest illustration of this uncertainty relates to forecasting asset class expected returns. For many risky assets, investors (out of necessity) rely largely on historical returns to develop expectations of return for those assets in the future. Even under the assumption that the true return-generating process is stable (a strong assumption, given that the dynamic nature of capital markets suggests that return distributions likely evolve through time), returns are inevitably estimated with error. There is noise in the return-generating process, and this noise leads to sampling error. Any given sample of historical data is likely to produce an incorrect estimate of expected return, even if the average across all possible samples accurately reflects the true expected return.

Performing any form of strict optimization is inherently dangerous, as the process attributes 100% certainty to the inputs; in reality, uncertainty is unavoidable. MVO is particularly susceptible to making decisions based on noise rather than true information, given its reliance on return expectations and the noise inherent in those estimates. And given its tendency to produce concentrated portfolios, it risks “putting all of its eggs in one basket” by foregoing diversification in the pursuit of higher returns that, in reality, are often illusory.

Our Experiment

Aware of this danger, we wanted to test the performance of MVO in the presence of uncertainty and to compare it to an alternative method of portfolio decision-making. To give MVO its fair chance, we sought to create an environment in which returns, risks and correlations were estimated with error but without bias (i.e., the estimates were incorrect in any given sample but were correct on average across all samples). While the nature of uncertainty makes it impossible to quantify precisely, we chose a method that allowed for a wide range of degrees of uncertainty and tested the different portfolio construction methods under various levels of sampling error.

To represent this uncertainty, we started with our 10-year capital market assumptions (CMAs – returns, risks and correlations) for U.S. large-cap stocks, U.S. core fixed income, and commodities.¹ Using these parameters as inputs and assuming a multivariate normal distribution, we simulated returns for each asset class over a given number of years.² Based on these simulations, sample returns, risks and correlations were calculated, and MVO was performed using these sample estimates. In addition, an MD portfolio was also constructed based on these sample (estimated) parameters by maximizing the expected diversification ratio (defined by Choueifaty and Coignard as the weighted average of assets’ standalone volatilities divided by the volatility of the portfolio as a whole³). These portfolios (which are constructed based on parameters that are estimated with error) were then tested on separate samples generated from the “true” returns, risks and correlations (SEI’s actual CMAs). The resulting expected Sharpe Ratios were compared to determine which method offered better risk-adjusted performance in the presence of uncertainty.

Diversification Ratio =

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\frac{\text{Weighted Average of Asset Class Standard Deviations}}{\text{Total Portfolio Standard Deviation}}
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It is important to understand two different factors influencing the results of this analysis. First, the CMA inputs obviously affect the outcomes associated with each method. The MVO method explicitly optimizes based on its return estimates, whereas the MD method uses only risks and correlations, implicitly assuming that each asset class offers the same risk-adjusted return. In some sense, this gives MVO a “head start” versus MD because it is able to capitalize on differences in true risk-adjusted returns. The size of this advantage will depend on the assumed variation in risk-adjusted returns across asset classes.

Second, the amount of noise, or sampling error, will also materially alter the results of each optimization. Various sets of simulations were run in which returns, risks and correlations were estimated based on different numbers of years of data. The more years of available data, the more the distributions will tend to converge to the true underlying distributions, and the less error will be present in the sample estimates. In this setting, less error corresponds to less uncertainty, and a higher degree of precision in estimating the true parameters. Given MVO’s sensitivity to estimation error, it follows intuitively that MVO will tend to perform better when it is given more data on which to base its estimates.

Results

Perhaps surprisingly, our results suggest that, given our CMAs, the presence of uncertainty can cause strict MVO to be suboptimal even from a mean-variance (i.e., Sharpe Ratio) perspective. The reason for this underperformance lies in the aforementioned vulnerabilities of MVO. When returns are estimated with a degree of error, MVO is susceptible to creating

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¹ It is important to note that these assumptions reflect the state of the current yield environment, meaning that our fixed-income return assumptions are quite conservative in penalizing bond returns for the current state of low yields and forecasted rises in interest rates over time.

² As explained later, the number of years was held constant within a given set of simulations, but various sets were produced with different numbers of years simulated in order to represent different levels of sampling error.

concentrated portfolios based on incorrect information. For instance, it will over-allocate to stocks when its stock return estimate is too high, and will under-allocate to stocks when its stock return estimate is too low, all else equal. In both cases, it will fail to take advantage of the full benefits of diversification and will make inefficient use of its risk budget, generating excessively concentrated portfolios rather than diversifying across all three asset classes (U.S. large-cap stocks, U.S. core fixed income and commodities) and fully capitalizing on their collective risk premia. In contrast, MD outperforms MVO in terms of Sharpe Ratio in the presence of meaningful uncertainty because it seeks diversification regardless of return expectations, and its method of optimization is more stable and less dependent on small differences in return estimates.

The figure below demonstrates the relationship between sampling error and out-of-sample performance across both methods. Again, fewer simulated years of data correspond to higher sampling error, and weaker relative performance of MVO versus MD. As the amount of available simulated data increases and estimation error declines, MVO closes the performance gap versus MD and eventually overtakes it in terms of risk-adjusted returns. This result is not surprising given the implicit assumption made by MD that risk-adjusted returns are equal across asset classes. At some point on the curve, the “head start” given to MVO via its ability to incorporate return estimates will overcome its increased susceptibility to estimation error. Clearly, the results depend on the amount of estimation error and on the assumed “true” risk-adjusted returns of each asset class. However, the example illustrates that, under what we consider to be a quite reasonable set of assumptions (indeed, the assumptions that we actually use), and in the presence of a moderate amount of uncertainty, diversification can defeat MVO at its own game of maximizing risk-adjusted returns.

**Conclusions**

The results of this analysis carry far-reaching implications for our investment process. We have communicated in the past that we do not follow a strict MVO methodology in constructing our portfolios. We think that sufficient uncertainty exists to render pure MVO potentially dangerous in practice (in the above framework, it is hard to think of any asset class for which 100 years of stable, reliable data exist). Again, while we are not advocating a particular approach to portfolio diversification, we remain confident that diversification, broadly construed, is the cornerstone of sensible portfolio construction. We are not so confident in our forecasts as to ignore the possibility that they are wrong, and would not risk our investors’ money on such an arrogant assumption.

However, our results also demonstrate why we do not pursue a strict “diversification” strategy that completely ignores return expectations. While data for certain asset classes might be scarce, we in the Portfolio Strategies Group spend our days considering relationships between asset classes and economic variables and developing well-researched assumptions as to the likely path of market outcomes over time.

Because we are able to develop forecasts based on both historical data and forward-looking economic intuition, we do believe that our assumptions for assets' risk-adjusted returns can and do add value to investors' portfolios. Consequently, we include both return expectations and diversification considerations in constructing portfolios, and we blend those inputs based on our relative confidence in both.

We certainly make short-term forecasts, and we hope to add value through rigorous research in this area, but we do not pretend to be infallible. Accordingly, we pursue meaningful diversification to protect investors from the inevitable uncertainty of capital markets. This is why we allocate to bonds even when our near-term fixed-income return forecasts are relatively low and why we include inflation hedges in our portfolios even when we do not expect inflation to explode in the immediate future. This desire for protection motivates us to seek broad diversification. Our analysis shows that such protection often does not reduce risk-adjusted returns and, in fact, can increase Sharpe Ratios in many instances. If diversification is the only free lunch in investing, consider us gluttons.
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